**Caution coupling explains linear phase of Covid-19 curves**

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The understanding of epidemics through modelling is gained both through highly simplified population models such as the early SIR model [1, 2] (distinguishing Susceptible, Infective, and Recovered individuals) and its extensions: both to more stage-differentiated models like SEIR [3-6] and SIDARTHE [7, 8] and age-differentiated social mixing models [9, 10], and to include stochastic effects [11, 12] and spatial [13-15] or other network structure [16-18]. Cause and effect can also be analysed in agent models [19, 20] making use of detailed population data (big data) [21, 22]. However, certain key features of the Covid-19 pandemic data affecting us all have not been explained simply. In particular, the remarkably common extended linear phase in countries’ cumulative confirmed cases is poorly understood. While small world network effects may account for non-exponential early growth [16], they do not explain the linear saturation effect well below levels required for “herd immunity”. In this letter, we show that central features of this societal response complexity can indeed be understood using the S(E)IR(D) family of models by means of a single additional deterministic coupling, one reflecting societal introspection: *caution*. As society reacts to and endeavours to contain the strong effects of the Covid-19 virus on health, social well-being and the economy, it may appear that explanation requires every aspect of society to be modelled in detail: from the use of public transport, religious practises and child play patterns to political decision making, the media and legislature[23]. However, the prolonged near linear response both in the cumulative number of Corona cases as well as those in individual populations can be understood as a generic human response to the epidemic in the age of information. This insight may in turn help us to focus societal measures and responses more effectively.

In March 2020, a study in Oxford [24] suggested, using an SIR model, that a much larger portion of society may be infected than normally inferred from the testing data, which was biased to symptom carriers. This was in-line with standard epidemiological modelling that relates the epidemic proliferation peak and subsequent decline to the advent of a significant fraction of the population becoming immune to the disease. Contemplating the early linear global data, see Fig. 1, the author was struck by the similarities with bacterial viral proliferation kinetics inside a single cell and in evolution experiments *in vitro*, which systematically show exponential growth giving way to linear growth in the absence of immunity [25, 26]. The fundamental reason for this is specific resource limitation, for example of the Qß viral replicase enzymes responsible for copying the RNA viral genome [26]. This is, like the nesting site limitation in bird populations, a straightforward thing: when most replicase enzymes are occupied in the copying process with a particular RNA genome, they are not available for proliferation of other RNA, and the throughput of copied RNA becomes a constant, limited by the throughput of such copying enzymes. The author wondered if a related mechanism could account for the observed linear growth phases for Covid-19 in societies shown in Fig. 1.

We add an endogenous informed human cautionary response to simple compartmentalized S(E)IR models in the following way. We split the population of susceptible individuals S into two classes S and Sc, the former acting unaffectedly and hence experiencing exposure to the virus at the initial rate, the latter exercising informed caution and hence reducing their exposure to the virus by a factor c0<1. We then identify the subpopulation C in the deterministic population model most indicative of the negative impact of the disease for the population, as the trigger for this response. For example, in the case of the simple SIR model, it is the class C=I of infected individuals, and more realistically in the SEI3R model, which adds an exposed class E of not yet infectious individuals and two additional classes I2 and I3 of infectious individuals (in hospital and in intensive care respectively), it is the class C=I3. Although one may expect that the number of deaths also dramatically influences the execution of caution, it is a cumulative category which does not reflect the current situation as does I or I3. In fact, the daily number of deaths would be an appropriate indicator, but this is simply proportional to I in the SIR model and I3 in the SEI3R model.

To capture the response to this cautionary trigger, we first employ a simple reaction mechanism: a reversible caution binding mechanism. Individuals observing the pandemic can be triggered to become cautious but after a while also shed this caution. So the model involves both a forward transition of unaffected susceptible individuals S to cautious individuals Sc, with a rate proportional to the product of their densities in the population (as in the law of mass action for chemical interactions), as well as a reverse transition from cautious individuals to individuals acting unaffectedly, proportional to the density of cautious individuals (resulting in a natural single exponential decay of caution in time). The mechanism and equations of two representative endogenous caution models are shown in Fig. 2. Note that in this first approach, we do not distinguish between cautious and non-cautious exposed or infected individuals I. We have tested the impact of making this distinction in a second set of models, also shown in Fig. 2, which share the prediction of a linear phase, with a stronger impact of caution. Once in hospital, we assume naturally that all individuals are exercising (or having exercised for them by hospital staff) a significant degree of caution. Further model distinctions are analysed in the SI.

An alternative threshold trigger response replaces the product law by a threshold activation rate of the form where is the difference between the endogeneous cautionary trigger signal introduced above and a threshold value (e.g. 25% of hospital ICUs being occupied) and describes the sharpness of the transition. While this may be required to fit accurately the sudden press amplified responses to reports of rising death tolls, it is not essential to explain the observed long linear response phases.

The generic linear phases of growth for the cumulative confirmed cases are surveyed in data taken from Johns Hopkins University database, using a rolling average over 7 days to remove prominent weekly variations in reporting and some fluctuations, in Fig 1A for 7 major European countries (Italy, Spain, Germany, France, UK, and for comparison Sweden and Turkey) as well as for the 7 countries (US, Brazil, India, Russia, South Africa, Mexico, Peru) with the most cases (on Aug 1 2020) in Fig 1B. Despite the differences in detail, all the comparatively mature responses in Europe demonstrate an extended linear phase of growth, which is markedly different from the herd immunity saturation common to the S(E)IR family of models. The saturation also takes place well before the percentage of infectives can cause any effective herd immunity. Although still in the pre-saturation phase for some (notably India and south Africa), the other largest Covid-19 epidemic countries in Fig. 1B also demonstrate an extended linear saturation response (two such phases with differing degrees of caution regulation are visible in the US data).

We solved the ordinary differential equations of each model, using the python modelling package GomPy (with its interface to SciPy) for convenience. Fig. 3 shows the range of epidemic responses in the number of daily cases for the caution extensions to the simple (SEIR) and hospital differentiated (SEI3R) models for representative variation of the three caution parameters about typical values. Note that the second of the caution extensions (S3EIRcolumns 2 and 4), which includes caution for exposed and infected individuals, strengthens the caution response and also predicts strong second wave phenomena for some parameter values. The range of response forms appears to cover the majority of forms seen for 194 countries in the JHU data. Note that in Fig. 3, showing daily not cumulative cases as in Fig. 1, the linear phase of growth corresponds to constant daily cases.

An example of fitting the caution model to the data for Germany (confirmed, recovered and deaths) by varying the three parameters affecting the cautionary response (as well as the starting point of the infection) is shown in Fig. 4. [We plan to extend this figure to include three other countries with different shaped responses.] Note that this fit was achieved with simple manual adjustment, without taking into account various complications in the data: the increase in testing which leads to an increasing percentage of cases being confirmed, delays in registration of statistics, geographic and age structuring of infection etc. It is not the purpose of this paper to generate more accurate fitted models taking all of these complications into account. Rather, we wish the reader to share the insight that the seeming complexity of the societal response to the Covid-19 pandemic can actually be better understood as belonging to a single simple phenomenon that distinguishes the Covid-19 pandemic from earlier epidemics : the ubiquitous internet mediated feedback of daily data collection inspiring a social distancing cautionary response in the population. A preliminary scan of the cumulative deaths in all monitored countries (JHU data) is shown in the supplementary material (together with simple fits from the cautionary model). Note that while the deaths data is deemed more reliable, since it is less dependent on incomplete Covid-19 testing, fitting just one quantity is less stringent than the example shown in Fig. 4, involving all three curves (confirmed, recovered and deaths).

This phenomenom is not restricted to European, US and Russian responses. Whereas some countries, with geographic advantages in isolation, by executing radical containment policies, have managed to reduce the daily new infection rate of the Covid-19 virus to a rather constant low level, which as predicted by our model is non-zero, the more generic growth response is a transition from an exponential (or possibly power law [16]) phase to a relatively constant rate of growth, often with 100s or 1000s of cases per day, in some cases after an initial overshoot. The explanatory power of our models is apparent in routinely replacing the generic feature of the SIR family of models – an infection peak following an exponential growth phase as limited by herd immunity – by an infection peak at orders of magnitude lower infection numbers, not related to herd immunity, and having a long often flat tail which does not involve exponential decay of the number of infections.

should be contrasted with The simplest generic structure of epidemics, captured also by the SIR model, is initial exponential growth tempered by the rise of so-called herd-immunity in the population.

We briefly address a potential critique of our result arising from limitations in the testing procedure. If the number of available tests is limited in a country, then this could result in a saturation in the number of confirmed cases per day, resulting in an apparent linear growth. Although such limitations have occurred and are visible for example in the data from …, we argue that our overall conclusions are independent of this effect. Firstly, overall testing levels in a significant number of countries have exceeded by a large factor the number of positive cases. Secondly, the linear trend also occurs in the number of deaths. Thirdly, the numbers of tests have been increasing in most countries and the linear trend persists.

The implications of these results are instructive. Firstly, it is no longer herd immunity but cautionary measures that is the primary limiter of spread of the disease: the latter have effect at much lower disease frequencies than herd immunity which would only be achieved much later after an enormous societal cost. Secondly, the strongly asymmetric peak responses to the daily case statistics and the linear population responses in cumulative case indicators (and in deaths) result from an active regulation of the degree of caution exercised in the population. As evidenced by the case of Sweden this is not entirely dependent on government legislation, which itself is in response to fears of repercussions based on severity indicators like the number of deaths, but also occurs naturally in the population, for example through the ubiquitous distribution of statistics concerning ICU cases. Depending on the timescale of relaxation of caution, the response can also involve second waves and longer-term oscillation. Thirdly, the natural human response, and as we have seen also government mediation of this apart from in a few nations, is not to maintain strict caution measures up until disease eradication. The successive relaxation of measures, even close to disease eradication results in very long periods of nearly constant case frequencies, consistent with the cautionary regulation process that we describe in this paper. Examples of this include Germany, Australia, South Korea, Switzerland and many other states.

**Figures**

A

A screenshot of a cell phone

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B

**Fig.1 Common linear phases of growth in individual countries.** **A)** The cumulative numbers of confirmed cases of Covid-19 in the larger European countries, including Turkey (similar size, 82 million) and Sweden (minimal government control, 10 million) as interesting cases for comparison having strong and weak central intervention. **B)** Same for seven countries with largest population epidemics in the world for Covid-19 as of Aug 1 2020. The USA and Brazil have not had a consistent or effective communication of information about the disease, but the response is still effectively piecewise linear as expected from the cautionary model, like the more dampening Russian response. Data is taken from John Hopkins University compilation and averaged over last 7 days to remove weekly oscillations in reporting. The data is fitted with a simple 3-segment piecewise linear fit. Note that second waves of infection, affecting Spain in A, and to a lesser extent France and Germany, which depart from linearity, are also predicted in the more differentiated SC3EI3R version of the cautionary model.

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| SIR |  |  |
| SCIR |  |  |
| SEIR |  |  |
| SCEIR |  |  |

**Fig.2 The mechanism and equations of the two simplest endogenous caution models.** The two standard epidemic models SIR and SEIR are extended in the simplest way to self-regulating models with caution SCIR and SCEIR, by distinguishing two classes of susceptibles S and Sc with the switch to caution prompted by high observed number of infections and with caution decaying exponentially with time. Cautioned susceptibles have a reduced rate of infection. In a next step, the exposed E and infected individuals I can also be distinguished by caution (see SI Fig S1) in SC2IR and SC3EIR models. Likewise, more differentiated standard models like the SEI3R model, with three stages of infected individuals (I1 non-hospitalized, I2 hospitalized and I3 critical care) can be extended to cautionary feedback via I3 as shown SI Fig S1.

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**Fig. 3 Effect of caution feedback for representative variation of the caution parameters**. The two underlying models SEIR (left half) and the more realistic SEI3R (right half) with distinguished hospitalization and ICU treatment of infected individuals are extended with caution in two ways: (i) (1st and 3rd column) only adding cautioned susceptibles as in fig. 2, and (ii) (columns 2 and 4) adding cautioned susceptibles, exposed and infected S,E,I for all non-hospitalized classes (assuming hospitalized cases are always cautioned). The three rows show the variations of the response with each of the three caution parameters (c0,c1,c2) about reference values (0.2,21,0.3) corresponding to (infectivity reduction factor, caution retention time in days, and sensitivity to ICU occupation fraction or equivalently to death rate, more precisely the fraction of ICUs occupied leading to transition to caution @ 1/day). The ICU fraction relative to population is a known country dependent parameter, set to a typical western value of 0.002 in this case. The vertical axis in all plots is the daily confirmed cases as a fraction of the population (allowing application to different regions or nations).

A close up of a map

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**Fig. 4 Fit to country data (Germany) for caution arising from critical societal burden in SC3EI3R model**. The data for the registered fraction of population with Covid-19 infection (green), recovered (blue) and deaths x10 (red) is shown with points (daily JHU data) and curves from simulation of ODEs according to the model SC3EI3R (see Fig. S1 for mechanism and equations). The second wave in Germany is predicted correctly for a ca. 2-month caution retention time. The fit was produced manually by varying the caution parameter c\_1 as well as the fraction of infected individuals (at start date Jan 25). The parameters of the fit are {c0: 0.1, c1: 0.015, c2: 104, I0: 10-6.73} and the fixed parameters assumed were { β1: 0.4, β2: 0, β3: 0,α: 0.2, γ1: 0.07, γ2: 0.167, γ3: 0.1, p1: 0.03, p2: 0.083,μ: 0.1} based on the standard parameters Exposure=0.4, IncubPeriod=5, DurMildInf=10, FracMild=0.7, FracSevere=0.20, FracCritical=0.1, CFR=0.05, TimeICUDeath=5, DurHosp=4, FracConfirmedDet=0.125, FracDeathsDet=1.0, ICUFrac= 0.001.

Comments: This figure needs to be replaced by a panel of 4 countries with fits. Choose:

# Supplementary Material

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| --- | --- | --- |
| SIR |  |  |
| SCIR |  |  |
| SC2IR |  |  |
| SEIR |  |  |
| SCEIR |  |  |
| SC3EIR |  |  |
| SEI3R |  |  |
| SCEI3R |  |  |
| SC3EI3R |  |  |

**Fig. S1 Mechanism of all models analysed in this work.** The three standard models SIR, SEIR and SEI3R are each extended with cautionary feedback in two ways: (i) firstly just distinguishing cautioned susceptibles and then (ii) distinguishing also cautioned exposed and/or infected individuals (non-hospitalized ones to be more precise). In each case the reference class employed to gauge the cautionary feedback is the class of most severely affected infected individuals. We assume for simplicity that the transition to caution is independent of exposure in the differentiated models.

Fig not yet available

**Fig. S2 Piecewise linear fits to all country data for cumulative deaths.** The piecewise linear analysis is extended to include all countries (up to Aug 1) for their cumulative deaths (deemed more reliable than the cumulative infections which depend more strongly on testing rates).

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**Fig. S3 Effect of caution feedback for representative variation of the caution parameters**. The three underlying models SIR (left third), SEIR (middle third) and the more realistic SEI3R (right third) with distinguished hospitalization and ICU treatment of infected individuals are extended with caution in two ways: (i) (1st and 3rd column) only adding cautioned susceptibles as in fig. 2, and (ii) (columns 2 and 4) adding cautioned susceptibles, exposed and infected S,E,I for all non-hospitalized classes, (assuming hospitalized cases are always cautioned). The three rows show the variations of the response with each of the three caution parameters (c0,c1,c2) about reference values (0.2,21,0.3) corresponding to (infectivity reduction factor, caution retention time in days, and sensitivity to ICU occupation fraction or equivalently to death rate, more precisely the fraction of ICUs occupied leading to transition to caution @ 1/day). The ICU fraction relative to population is a known country dependent parameter, set to a typical western value of 0.002 in this case. The vertical axis in all plots is the daily confirmed cases as a fraction of the population (allowing application to different regions or nations).

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